CHAPTER 5

Returns, Risk, and Financial Due Diligence*

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Returns are the prospective financial rewards from investment. Risk is the potential for fluctuations in returns to engender losses. If investors are risk averse—as most appear to be—then they should demand higher expected returns from riskier investments.

In the wake of the credit crisis and the Bernard Madoff scandal, investors and regulators are clamoring for more rigorous *financial due diligence* by fund managers, institutional investors, and other market participants. Financial due diligence is the process by which investors try to ascertain, among other things, the potential risks and returns of a contemplated investment.

Both qualitative and quantitative methods are used to determine whether a given investment offers a fair risk/return trade-off (and what that trade-off is). Due diligence analysts use qualitative methods to examine hard-to-quantify variables—for example, portfolio manager reputation, internal control quality, reporting adequacy, and regulatory compliance. Investors employ quantitative methods to examine matters that more naturally lend themselves to empirical analyses—especially the risk and return characteristics of contemplated investments.[1]

Some potential investments can appear undesirable until the due diligence analyst properly measures their risks and returns, at which point the investment may seem more attractive. Alternatively, other potential investments can look

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appealing until the due diligence analyst appropriately analyzes risks and returns and determines that the investment is unpalatable.

Part of the process of identifying investments with fair risk/return trade-offs includes spotting investments that seem too good to be true. As Judge Richard Posner observed in a Ponzi scheme case, "Only a very foolish, very naive, very greedy, or very Machiavellian investor would jump at a chance to obtain a return on his passive investment of 10 to 20 percent a month (the Machiavellian being the one who plans to get out early, pocketing his winnings, before the Ponzi scheme collapses). It should be obvious that such returns are not available to passive investors in any known market, save from the operation of luck."[2] Financial due diligence helps investors avoid becoming one of those "very foolish, very naive, very greedy, or very Machiavellian investor[s]" that Judge Posner and other actors in the courts look for in such situations.

In this chapter, we first explain basic concepts of risk and return in financial economics with an eye toward the task of financial due diligence. We then illustrate the applications of these concepts in financial due diligence using the example of Bernard Madoff Investment Securities.

BASIC CONCEPTS OF RISK AND RETURN IN FINANCIAL ECONOMICS

The *return* on an asset over some period of time (returns are always relative to some time period, whether an instant, day, month, year, etc.) is its payoff over that time period relative to its initial value (i.e., the value of the asset at the beginning of the period). We summarize some of the most popular ways of measuring returns in Appendix A. Most generally, the net return on a financial asset from time t to $t + 1$ is

$$r_{t+1} = \frac{x_{t+1} - p_t}{p_t} = \frac{d_{t+1} + p_{t+1} - p_t}{p_t} \tag{5.1}$$

where p_t is the price of the asset at time t
x_{t+1} is the payoff to investors at time $t + 1$
d_{t+1} reflects distributions to investors (e.g., dividends or interest) at time $t + 1$
p_{t+1} is the price of the asset or portfolio at the end of the holding period

The *risk* of an asset is the potential for returns to fluctuate unexpectedly. Returns vary for a number of reasons, including, but not limited to, changes in prices and interest rates (market risk), the nonperformance of counterparties or obligors (credit risk), cash flow shortfalls (funding risk), and forced liquidations of losing positions at unreasonable prices or spreads (liquidity risk).

A key premise of modern financial economics is that return and risk are related—in particular, investors expect a higher return for bearing higher risk. When an asset pays off a known amount with certainty, that asset is called *risk-free.* Competition in the market for risk-free assets will force the rate payable on riskless assets to the risk-free rate.[3]

Excess Returns and Alpha

Risk-averse investors will demand a return in excess of the risk-free rate to compensate them for bearing risks they prefer to avoid. Risks to which investors are averse are risks that lead to losses—so-called *downside* risks. Some investors are content with low returns as long as they face limited downside risk. Others are willing to bear more downside risk in the pursuit of higher returns.

In theory, only downside risks that investors cannot eliminate by diversification should earn higher expected returns. Such risks are called *systematic* risks. Because no investor can eliminate systematic risk simply by adding other assets with systematic risk to a diversified portfolio, the asset must offer a return commensurate with its systematic risk to persuade the investor to hold the asset.

Risks that the investor can eliminate by holding the asset in a diversified portfolio, by contrast, are called *idiosyncratic* risks. In equilibrium, investors should not earn a return for bearing idiosyncratic risk, which *is* diversifiable by most investors.[4] Otherwise, all investors would have an incentive to add any asset offering a return for idiosyncratic risk to their already diversified portfolio. The idiosyncratic risk would disappear in the portfolio, leaving only the return. Such free lunches cannot survive in competitive capital markets.

Much research in financial economics aims at understanding the risks for which investors demand compensation in capital markets. That is, financial economists seek to understand the sources of systematic risk and the returns that investors demand for bearing those risks. If we could measure systematic risk perfectly, we then could estimate the expected return actually being offered by the asset, $E(r)$, and compare it to the expected return $E(r*)$ that compensates for the asset's systematic risk. The difference, if any, between the two is known as alpha:

$$\alpha = E(r) - E(r^*)$$

A zero or negative alpha indicates that the investment is just compensating or undercompensating investors for the risks that affect the underlying payout on the security or portfolio. But if alpha is positive, the investment is overperforming relative to its measured risks. That is the reason many investors claim to "seek alpha"—investments with positive alpha are offering expected returns that more than compensate for their risk.

How much return an asset should pay to compensate for its systematic risk depends on the sources of systematic risk, the exposure of the asset to those sources, and the premiums that investors demand for bearing that risk. Answering those questions requires a model of market equilibrium for capital assets, often referred to as *asset pricing models*. Different asset pricing models will, in general, assume the existence of different sources of systematic risk and thus typically give rise to different estimates of $E(r^*)$.[5]

The Capital Asset Pricing Model

The capital asset pricing model (CAPM) is the simplest and best-known theoretical asset pricing model. In the CAPM, the only source of systematic risk is the extent to which an asset's return moves together (covaries) with the return of the weighted average of all other assets, where the weights are the market values of all of the other assets in the world.

The idea is fairly simple. Suppose that you could buy a little bit of every asset in the world and that your own personal portfolio had the same rate of return as the weighted average of all assets in the world—that is, your own portfolio of risky assets is just a tiny version of the whole portfolio of world wealth. Suppose further that you prefer more money to less but that, at your current wealth, the pain of losing a dollar hurts more than the pleasure of gaining a dollar feels good.

Now consider any one asset in the world. If the asset performs well (earns good returns) when all of your other assets are doing well, that is doubtless a good thing. But the problem is that you are earning money from that asset when you are already earning money on everything else. And it works the other way. If that asset is moving with the rest of your wealth then it is going to perform poorly when the rest of your assets also are doing poorly. That's not good. So the more an asset's return covaries with the rest of the wealth in the world, the more you are going to want to get paid to hold that asset—that is, the higher the expected return you will demand.

In the CAPM, the systematic risk is the strength of the covariance between the returns on a given asset and the returns to the rest of the wealth in the world. That is, the CAPM return that investors can expect on some asset or portfolio j, $E(r_j^*)$, is related to its systematic risk as follows:

$$E(r_j^*) - r_f = \beta_j[E(r_m) - r_f] \tag{5.2}$$

where r_j is the return on asset or portfolio j
r_m is the return on the market portfolio of world-invested wealth
r_f is the risk-free rate
β_j is a measure of the extent to which the returns r_j and r_m move together—namely, the coefficient in a regression of asset j's excess returns on the market's excess returns

or

$$\beta_j = \frac{Cov(r_j, r_m)}{Var(r_m)}$$

The only source of systematic risk in the CAPM—and the only thing driving differences in expected returns given r_m and r_f—is the asset's β.

To determine whether there is any alpha, we take a sample of N historical returns on a portfolio j and run the following regression:

$$r_{j,t} - r_f = \alpha + \beta_j[r_{m,t} - r_f] + \varepsilon_{j,t} \tag{5.3}$$

$$for\ t = 1, \ldots, N$$

If the asset earned returns that compensated for its CAPM risk and the CAPM correctly models asset returns (this is an important assumption), then the estimated intercept α in regression (3) should be zero. A positive estimated alpha is evidence that the asset earned more on average than the CAPM predicted.

Interpretations of positive estimated alphas can be challenging. If the CAPM is true, the estimated alpha is good evidence of positive abnormal returns—in other words, an investment that beat the market. But it is much more difficult to interpret the positive alpha if the CAPM is not a good description of asset pricing. In that case, the positive alpha may be due entirely to the omission of some other risks for which the investors holding the asset were compensated but which is not reflected in the CAPM. The asset will have earned higher average returns than the CAPM predicted not because of any mispricing that reflected the opportunity for returns above those necessary to compensate for risk, but instead because those returns compensated for sources of risk omitted from the CAPM.

Other Asset Pricing Models

Much empirical evidence suggests that the CAPM does not adequately capture all sources of systematic risk in asset returns. The co-movement of returns with other variables helps explain these deviations from the CAPM. Asset pricing models that include these variables often characterize expected excess returns as

$$E(r_j^*) - r_f = \beta_{1,j}\delta_1 + \beta_{2,j}\delta_2 + \cdots + \beta_{k,j}\delta_k \tag{5.4}$$

where $\beta_{k,j}$ is the kth regression coefficient of asset j's excess return on the kth risk factor
δ_k is the risk premium of the kth risk factor

The risk factors are proxies for economic variables with which investors are concerned in defining good and bad times. In the CAPM, the only such risk factor was co-movement with the market.

A currently popular version of the general model shown in equation (5.4) is the Fama and French (1993) three-factor model, which describes expected excess returns on an asset or portfolio j in terms of three systematic risk factors:

$$E(r_j^*) - r_f = \beta_m\delta_m + \beta_{SMB}\delta_{SMB} + \beta_{HML}\delta_{HML} \tag{5.5}$$

where δ_m is the excess return on the market (the same factor used in the CAPM)
δ_{SMB} is a variable formed from the difference in returns to big versus small market capitalization stocks (designed to capture the observed factor of firm size in explaining differences in average returns across stocks)
δ_{HML} is a variable formed from the difference in returns on stock with high versus low book-to-market ratios (designed to capture the observed explanatory factor of such measures in explaining differences in average returns across stocks)

The various β's are the respective regression coefficients. A four-factor version of the model also includes a variable designed to capture the tendency of recent good and bad performance to continue, known as the *momentum effect*.

Like the CAPM, running a regression of the form in equation (5.5) generates an estimated intercept that should be zero if the Fama-French model is a true representation of the relation between expected excess returns and systematic risk. A positive estimated intercept indicates that the average return of the asset or portfolio exceeds the risk-free rate by more than the systematic risk premium. Also like the CAPM, the positive intercept may reflect abnormal performance unexplained by risk or, alternatively, misspecification of the asset pricing model that has omitted proxies for the true sources of systematic risk.

Measures of Total Risk

To augment or obviate the search for an appropriate asset pricing model to estimate alpha, many analysts also employ measures of returns relative to some measure of total risk that does not attempt to decompose return fluctuations into systematic and idiosyncratic components. One such measure is the Sharpe ratio:[6]

$$SR_j = \frac{\bar{r}_j - r_f}{\sigma_j}$$

where SR_j is the Sharpe ratio on asset or portfolio j
$\bar{r}_j$ is the average return on asset or portfolio j
σ_j is the volatility of returns on that asset or portfolio

Volatility is often estimated as the standard deviation of returns over an historical period, perhaps using rolling moving averages or more structured models of the evolution of volatility over time.[7]

A problem with using the Sharpe ratio for financial due diligence, however, is its measurement of risk using only the volatility of excess returns. Volatility is a symmetric measure of risk that reflects deviations both above and below average returns. But if the true return distribution is negatively skewed or fat-tailed, volatility is an incomplete description of return dispersion. And, as noted earlier, it is the downside risk with which most investors are more concerned.

Consider, for example, a portfolio that consists of short positions in out-of-the-money equity put options. The portfolio earns a premium as long as stock prices do not decline significantly. But if stock prices collapse, the options move into-the-money and the value of the portfolio crashes. Yet the volatility of the payoff on the short option portfolio is lower than the volatility of a similar portfolio invested in the stocks underlying the puts. In both portfolios, investors lose when share prices decline. But in the stock portfolio, investors make money when prices rise, unlike the option portfolio in which the maximum payoff is the premium collected. The distribution of payoffs on the option portfolio thus is truncated, which reduces the estimated volatility of returns. That lower volatility, however, results from chopping off the potential *upside* of the strategy. Volatility thus has been reduced at the expense of negative skewness and fat tails in the payoff distribution. As such, it is by no means clear that the option portfolio is less risky than the stock portfolio even though the returns on the former are less volatile than on the latter.

To measure the risk/return ratio for an asset or portfolio with skewed and/or fat-tailed returns, an analyst may instead evaluate average excess return relative to an estimate of downside risk (DSR). Unlike volatility, DSR measures the risk of only those returns below the average or some target. The analogue of the Sharpe ratio for measuring average excess returns per unit of DSR is the Sortino ratio:

$$Sortino\ Ratio = \frac{\bar{r} - r_f}{DSR}$$

Quite a few different ways of measuring DSR can be used to calculate the Sortino ratio. One such measure, the downside semi-standard deviation (DSSD), is defined as

$$DSSD = \sqrt{\frac{1}{M} \sum_{\{s,t,r_t<\bar{r}\}\ 1}^{M} (\bar{r} - r_t)^2}$$

where M is the number of returns in the sample below the average return. DSSD thus measures the so-called bad part of the standard deviation. If the underlying return distribution has a fat left-hand tail, the DSSD provides a better measure of risk than volatility.

Another popular measure of DSR is value at risk (VaR). For an estimated distribution of potential returns, VaR measures the return threshold that the investor expects to exceed $(1 - X)$ percent of the time, where X is usually set at 1 percent or 5 percent. A 99 percent monthly VaR of –15 percent, for example, means that the portfolio is expected to generate monthly returns below –15 percent only 1 percent of the time. The underlying return distribution used to compute VaR can be generated parametrically, nonparametrically, by simulation analysis, or with some mixture of those methods.[8]

A significant drawback of VaR is that it does not tell us the magnitude of potential losses below the critical level. A 99 percent monthly VaR of –15 percent suggests that returns should not be below –15 percent more than 1 percent of the time, but it does *not* tell us whether the 1 percent of violations consist of, say, –16 percent returns or –1,600 percent returns. To address this, market participants sometimes define VaR in terms of conditional expected loss, otherwise known as *tail VaR* or *t-VaR*.

Analysts typically compare a calculated risk/return ratio with the risk/return profile of similar assets. For example, Exhibit 5.1 shows historical return, risk, and return/risk ratios for the CRSP Value-Weighted Portfolio of NYSE, NASDAQ, and AMEX stocks from 1947 to 2008. All of the measures of return relative to risk are below 0.50. The definition of risk, moreover, changes the results noticeably. The Sortino ratio using 95th percentile VaR as a measure of DSR, for example, is appreciably lower than the Sharpe ratio.

Exhibit 5.1 Risk and Return Statistics on the CRSP Value-Weighted Portfolio of NYSE, NASDAQ, and AMEX Stocks, 1947 to 2008

	Monthly	Annual
Returns:		
Average Market[a] Return	0.924%	11.981%
Average 30-day T-Bill Return	0.377%	4.659%
Average Excess Return[b]	0.546%	7.323%
Risk:		
Volatility of Excess Returns	4.251%	18.149%
DSSD of Excess Returns	4.723%	20.897%
95th Percentile VaR of Excess Returns[c]	6.513%	22.293%
Return/Risk Ratios:		
Sharpe Ratio	0.1286	0.4035
Sortino Ratio (DSSD)	0.1157	0.3504
Sortino Ratio (VaR)	0.0839	0.3285

[a]CRSP Value-Weighted Portfolio (including distributions).
[b]Average market return minus average 30-day T-bill return.
[c]Absolute value of fifth percentile excess return.
Source: Center for Research in Security Prices.

PUTTING THEORY INTO PRACTICE—THE MADOFF EXAMPLE

We now illustrate the application of risk/return analysis to financial due diligence by examining the detection of a Ponzi scheme. In a Ponzi scheme, the promoter solicits funds from customers for investment in some portfolio or strategy, but little or no investing actually occurs. Redemption requests and distributions are financed by cash received from new participants in the scheme—robbing Peter to pay Paul, as it were—and the remaining cash is distributed to the participants in the fraud.

Because almost all investment managers who appear to have abnormally high average returns will attribute their results to skill, the self-reported performance explanations of investment managers are likely to hold little weight in the due diligence analysis. Investment managers do not, after all, self-proclaim their fraudulent investments.

The problem is especially difficult when Ponzi schemes do not promise super-high returns. When long-run average returns are just high enough to be enticing but not so high as to be obviously unrealistic, then the tools we have discussed thus far can be valuable components of the due diligence process. We illustrate using the case of Bernard L. Madoff Investment Securities ("Madoff" in the pages that follow).

Madoff's Ponzi Scheme

In the largest investor fraud by an individual in history, Madoff primarily marketed a single investment strategy—known as a *split strike conversion*—in which he claimed to be purchasing blue-chip stocks in the S&P100 Index and simultaneously selling out-of-the-money calls and buying out-of-the-money puts on the S&P 100

Index. Normal enough in its own right, a split strike conversion strategy is essentially just a stock index arbitrage program and, as such, should have relatively low risk and generate modest returns.

Yet Madoff boasted average returns of nearly 10.5 percent per annum for the 17 years during which the Ponzi scheme went undetected. Even when the market fell nearly 40 percent through November 2008, Madoff was still reporting a positive 5.6 percent year-to-date return (Applebaum et al. 2008).

Ponzi schemes generally fall apart when larger-than-expected redemptions occur. But that never happened with Madoff. If not for the collapse of equities during the credit crisis, Madoff's fraud might have remained undiscovered for many more years. Madoff's scheme apparently went undetected for so long part because it was an *affinity fraud* aimed at the wealthy Jewish community in New York and Palm Beach. Within that community, Madoff was a well-known figure with impeccable references; his investors trusted him. Indeed, some within Madoff's target affinity group report having tried to invest with him but having been turned away—no doubt adding to his appeal.[9] In addition, Madoff's returns were generally not so high as to be completely ridiculous on their face.

A Risk/Return Analysis of a Madoff Feeder Fund

Most of Madoff's money came from feeder funds that secured investments from customers and then used Madoff as either the investment manager or broker. To analyze the risk and return of Madoff's scam, we obtained returns from July 1989 through December 2000 on one of Madoff's largest feeder funds.[10] Although some of Madoff's feeder funds had other investments, we understand that the fund we examined was invested almost exclusively with Madoff.

Alpha

Exhibit 5.2 shows Madoff's estimated alpha from the CAPM and the Fama-French model regressions—equations (2) and (5), respectively. If we run a CAPM regression of the feeder fund's excess returns on the market portfolio's excess returns, we get a statistically significant estimated α of 0.7251 percent per month. In the naïve CAPM world, it looks like Madoff was earning about 75 basis points per month above the return commensurate with the systematic risk of the market. The Fama-French regression yields a similar estimate of 0.7209 percent per month; adding the two additional proxies for systematic risk only reduces average returns by about half a basis point per month.

Using both the CAPM and Fama-French models, it appears as though Madoff's feeder fund was adding significant value in excess of the systematic risk of the fund. As noted earlier, these positive alpha estimates could be the result of model misspecification. But part of the due diligence process is identifying red flags like this one and following up with additional qualitative and quantitative analysis.

Returns Relative to Total Risk Measures

Exhibit 5.3 shows monthly returns from July 1989 through December 2000 on the CRSP Value-Weighted Portfolio compared to Madoff's monthly returns. The average monthly return on Madoff was 1.18 percent, as compared to an average monthly return on the market of 1.24 percent over this period. As Exhibit 5.3 also

Exhibit 5.2 Alpha Regressions of Madoff Feeder Fund Returns

	CAPM	Fama-French
α	0.007251**	0.007209**
β_m	0.053397**	0.053783**
β_{SMB}	n/a	−0.04799*
β_{HML}	n/a	−0.01809
N	138	138
R^2	0.0727	0.1147

*$p < 5\%$.
**$p < 1\%$.

shows, however, Madoff's returns exhibited very low volatility—0.83 percent per month as compared to 4.08 percent per month for the market.

Despite average returns slightly below the market, the Sharpe and Sortino ratios for Madoff are well above the market, as shown in Exhibit 5.4. The Sharpe ratio over this period was 0.9516 for Madoff, as compared to 0.2028 for the market. And Madoff's Sortino ratios (measured with DSSD and VaR, respectively) were 1.0730 and 2.9515, compared to the market Sortino ratios of 0.1741 and 0.1465.

Moving down the rows in Exhibit 5.4, average returns are divided by increasingly conservative measures of risk. As expected, the risk/return ratios for the market decline as the measure of risk in the denominator increases. But the

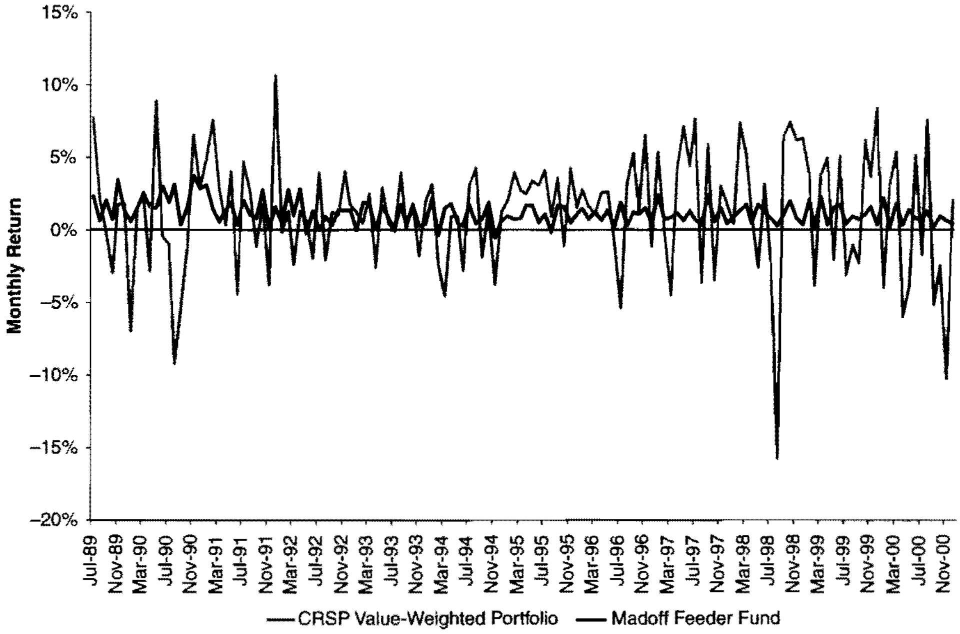

Exhibit 5.3 Monthly Returns on the Market versus Madoff Feeder Funds, July 1989 through December 2000
Source: Center for Research in Security Prices.

Exhibit 5.4 Sharpe and Sortino Rations, Madoff versus CRSP Value-Weighted Portfolio, July 1989 through December 2000

	Madoff	Market
Sharpe Ratio	0.9516	0.2028
Sortino Ratio (DSSD)	1.0730	0.1741
Sortino Ratio (VaR)	2.9515	0.1465

Madoff portfolio shows the opposite pattern—*increasing* risk/return ratios for progressively more conservative measures of risk. That indicates extremely thin tails in Madoff's return distribution vis-à-vis the market. In other words, not only do Madoff's returns exhibit little variation around the average, they also include few bad months.

Both the levels of the risk/return ratios and the thin-tailed distributions they indicate represent additional red flags. Although fraud is not the only possible explanation for the patterns in Exhibit 5.4, the data indicate that further due diligence is likely warranted.

Persistence and Serial Correlation

Another indicator of potentially too-good-to-be-true investments is excessive persistence in returns. Efficient capital markets are generally thought to follow close to random walks, especially over holding periods of a month or longer. As such, significant persistence in returns is a red flag to ask additional questions about why the performance of an asset or portfolio is seemingly so stable over time.

Return persistence is measured statistically by looking at serial correlation (aka autocorrelation). Specifically, we can run the following regression:

$$r_{t+1} = \rho_0 + \sum_{k=1}^{q} \rho_k r_{t-k} + \varepsilon_{t+1}$$

where q is the number of lagged returns that we want to examine. The regression coefficient ρ_k is the partial autocorrelation of returns at the kth lag. If returns fluctuate randomly, ρ_k should be zero at all lags. Positive estimated autocorrelations indicate persistence in returns—that is, an unusual high return in one period is likely to be followed by an unusually higher return in the next period.

The number of autocorrelation lags that an investor should examine depends on the frequency of available mark-to-market returns and the quality of that data. In annual returns, statistically significant positive autocorrelations at the one-year lag should be enough to raise an eyebrow. With monthly returns, looking at two or three lags is probably adequate.

Positive autocorrelation also reduces estimated volatility. Returns that exhibit persistence thus will tend to be less volatile and to have higher Sharpe ratios than returns following a random walk. The greater the persistence of returns, the lower is the estimated volatility of returns and the higher the Sharpe ratio. So even if positive autocorrelations show up for good reasons, further due diligence still may well be indicated.[11]

The first three partial autocorrelation coefficients on market portfolio returns are all statistically indistinguishable from zero, just as we would expect. But for Madoff, the partial autocorrelations are –0.19, 0.24, and 0.19 for the first three lags, all of which are statistically significant.

The positive autocorrelation on the second and third lags show persistence in returns that might be expected from a Ponzi scheme. Although returns persistence can be generated by infrequent marking to market of the underlying securities, Madoff's supposed focus on highly liquid S&P 100 stocks and options suggests that those autocorrelations cannot be explained by nonsynchronous trading or illiquidity alone.

The estimated autocorrelation at the first lag, however, is negative. That is more traditionally associated with phenomena such as market overreactions or prices that bounce between bids and offers. The same thing would also be consistent with a fictional pricing scheme that took average prices and then marked them up one month and down the next. But the explanation is not immediately obvious from the data.

So once again, we have a potential red flag—but only a potential one. Although the autocorrelations in the Madoff fund are consistent with a fictional-price Ponzi scheme, there are other explanations for these estimates. The autocorrelations thus are not conclusive on their own but should be the catalyst for asking additional questions.

CONCLUSION

In theory, identifying opportunities that are seemingly too good to be true can be accomplished by looking for abnormally high alphas. The problem, of course, is that the appearance of uncharacteristically high average excess returns may arise for different reasons: (1) the investment manager or trader is engaged in willful deception or fraud; (2) the investment manager or trader is pursuing authorized and legitimate investments but the measurement does not provide a true picture of risk and return due to errors in data or methodology; (3) the investment manager has been lucky; or (4) the investment manager has genuine skill. But in practice, the seemingly insurmountable empirical difficulties in testing asset pricing models makes it virtually impossible to distinguish between alphas that are *actually* positive and positive alpha *estimates* that are positive because of a misspecified asset pricing model.

In the Madoff example, warning signs were present in the data as of late 2000. But even with the benefit of hindsight, those warning signs were not unambiguously indicative of fraud in and of themselves. Nevertheless, the warning signs were sufficient to indicate that additional analysis—both quantitative and qualitative—may well have been warranted.

APPENDIX A: COMMON DEFINITIONS OF RETURN

A return is the payoff on a financial asset or portfolio relative to the initial value of that investment. Returns generally can be measured in one of three ways: discrete holding period returns, continuously compounded returns, or investment accounting returns.

Discrete Holding Period Returns

A holding period return is the return on an investment over some period of time during which the investor is presumed to hold the asset. The two most basic measures of holding period returns are gross and net per-period returns:

$$R_{t,t+1} = \frac{d_{t+1} + p_{t+1}}{p_t}$$

$$r_{t,t+1} = \frac{d_{t+1} + p_{t+1} - p_t}{p_t} = R_{t,t+1} - 1$$

where p_t is the time t price of the asset and d_{t+1} reflects any distributions to the investor such as dividends or interest payments.[12]

We also often want to know the effective N-period return on an asset, assuming the payoff on the asset is reinvested at the end of each holding period successively for N periods. An investment of $1 at time t that is rolled over for N periods yields a time $t + N$ value of

$$V_{t+N} = \prod_{j=1}^{N} (1 + r_{t+j-1,t+j})$$

The effective return over N periods is then calculated as

$$r_{t,t+N} = V_{t+N}^{\frac{1}{N}} - 1$$

$$r_{t,t+N} = [(1 + r_{t,t+1})(1 + r_{t,t+1}) \cdots (1 + r_{t+N-1,t+N})]^{\frac{1}{N}} - 1$$

$$r_{t,t+N} = [R_{t,t+1} R_{t+1,t+2} \cdots (R_{t+N-1,t+N}]^{\frac{1}{N}} - 1$$

Careful attention must be paid to the presumed compounding frequency in multiperiod return calculations. In general, an asset whose return is compounded q times per year over N years has an N-year effective holding period return of

$$r_{t,t+N} = q \left[\left(\frac{V_{t+N}}{V_t} \right)^{\frac{1}{qN}} - 1 \right]$$

where

$$V_{t+N} = V_t \prod_{j=1}^{qN} \left(1 + \frac{r_{t+j-1,t+j}}{q} \right)$$

Continuously Compounded Returns

A continuously compounded return is the *instantaneous* return on an investment, assuming that all distributions are continuously reinvested. In general, the continuously compounded return (aka geometric return) can be calculated from the corresponding holding period return r as follows:

$$r^{cc} = ln(1 + r)$$

In practice, continuously compounded returns are often computed as the log difference in prices between two periods. An N-period geometric return, for example is

$$r^{cc} = ln\left(\frac{p_{t+N}}{p_t}\right)$$

Investment Accounting Returns

Investment managers must calculate returns to conform to regulations or guidelines promulgated by supervisors and accounting organizations. Such investment accounting measures of return are often more difficult to calculate than holding period returns because they must take into account any contributions or withdrawals.

The ideal investment accounting measure is a true time-weighted return—essentially a holding period return in which individual holding periods are defined as trading days. At the end of any day t the value of the portfolio is defined as

$$V_t^e = p_t + d_t$$

where p_t is the mark-to-market value of the portfolio at the end of day t and d_t reflects any income or distributions on day t. The value of the portfolio at the beginning of day t is

$$V_t^b = V_{t-1}^e + C_{t-1}$$

where C_t reflects any cash withdrawals or contributions at the end of prior holding period $t - 1$. The time-weighted gross return over day t then is just

$$R_t^{TWR} = \frac{V_t^e}{V_t^b}$$

and the N-period net holding period return is

$$r_{t,t+N}^{TWR} = \left(\prod_{j=0}^{N} R_{t+j}^{TWR}\right) - 1$$

The principal reason that a true time-weighted return requires daily holding periods is that cash contributions and withdrawals may occur at any time. A manager thus needs to know values and returns on each day in order to account for cash distributions properly.

Many portfolio managers, however, do not have access to daily mark-to-market prices or are concerned about the quality of daily prices on illiquid positions. As an alternative, investors often compute approximate time-weighted returns (often misleadingly referred to as dollar-weighted returns) using the Modified Dietz method in which the N-period return is approximated as

$$r_{t,t+N}^{MDietz} = \frac{V_{t+N} - V_t - \sum_{j=0}^{N} C_{t+j}}{V_t + \sum_{j=0}^{N} C_{t+j}\tau_{t+j}}$$

where C_j is any cash withdrawal or contribution on date j, and

$$\tau_{t+j} = \frac{\tau_{t,t+N} - \tau_{t,t+j}}{\tau_{t,t+N}} \tag{5.6}$$

where $\tau_{t,t+j}$ is the total number of days in the holding period from t to $t + j$.

Finally, some investment managers compute a naïve dollar-weighted return on a portfolio as follows:

$$r_{t,t+N}^{DWR} = \frac{V_{t+N} - V_t}{V_t}$$

where

$$V_{t+N} = \sum_{k=1}^{K} C_k(1 + \upsilon k)^{\tau_k}$$

where K is the total number of days on which a contribution or withdrawal occurred in the holding period from t to $t + N$

k is an index variable indicating each of those withdrawal dates

τ_k is as defined in equation (5.6)

υ_k is the internal rate of return on the portfolio at k

NOTES

1. Not all market participants and due diligence analysts are created equal. Most of our comments here are intended to apply to relatively sophisticated institutional investors.
2. *Scholes v. Lehman*, 56 F.3d 750, 760 (7th Cir. 1995).
3. The risk-free rate will differ depending on the timing of the certain payoff, of course. The risk-free rate for an investment that pays off in a year will in general be different from the risk-free rate for an investment that pays off in two years, and so on.
4. There are some exceptions, most of which owe to capital market frictions. For a discussion, see Cochrane and Culp (2003).
5. For a review of the main asset pricing models, see Cochrane (2005).
6. See Sharpe (1966 and 1994).
7. Because we are really interested in knowing what the risk of an asset or portfolio *will be* and not what it *was*, measures of risk in the Sharpe ratio can be even more useful

when based on estimates of expected future volatility reflected in market prices. Option-implied volatility, for example, is a forward-looking estimate of volatility.

8. One popular method of measuring VaR, known as the *parametric normal* method, uses volatility to compute VaR. As a scaled measure of standard deviation, this does not add much to risk estimates that rely on volatility directly. But this is just one possible way to measure VaR. In general, VaR can also be measured in ways that do not rely exclusively on volatility and that allow for skewed and fat-tailed return distributions. See, for example, Culp (2001).
9. See Biggs (2009).
10. Subsequent references to Madoff's performance refer to this single feeder fund. We are grateful to Andy Lo for providing us with the feeder fund return data.
11. Various methods are available to adjust Sharpe ratios and other performance measures for autocorrelation that arises when hedge funds and private equity funds engage in *return smoothing* (for either legitimate or questionable purposes). For a good discussion, see Getmansky, Lo, and Makarov (2004) and Lo (2001 and 2008).
12. If distributions are paid before the end of the holding period, they can easily be restated to time $t + 1$ values.

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